UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Transients and Oscillations in RLC Circuits

Physics 401, Spring 2019.

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Transients and Oscillations in RLC Circuits. Outline

- **Transients. Definition.**
- **Transients in RLC**
- **Resonance in RLC**
- **Data analysis. Origin. Fitting.**

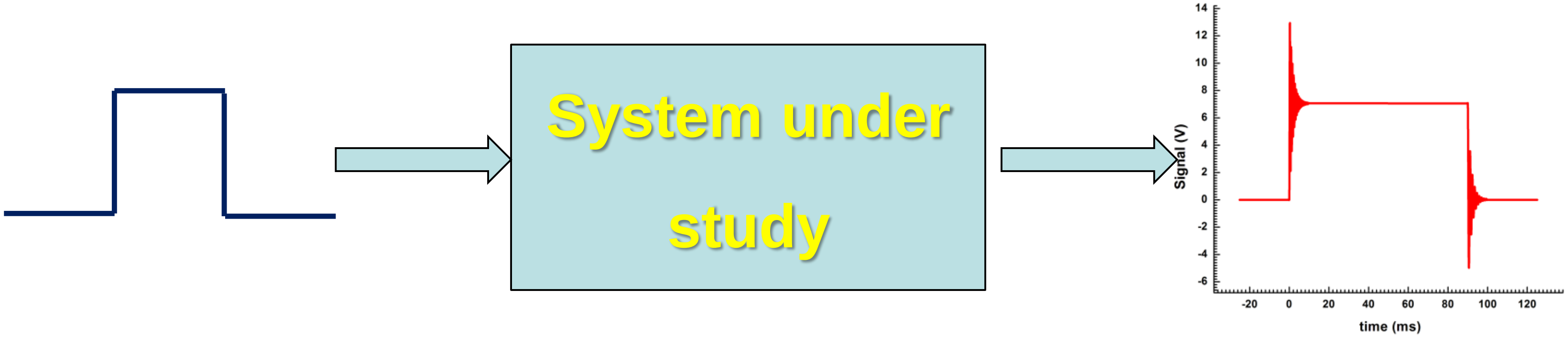
Main goals of this week Lab:

- To understand what are the transients in general
- Transients in RLC circuits. Different regimes of dumping
- Data analysis using Origin software

Transients. Definition.

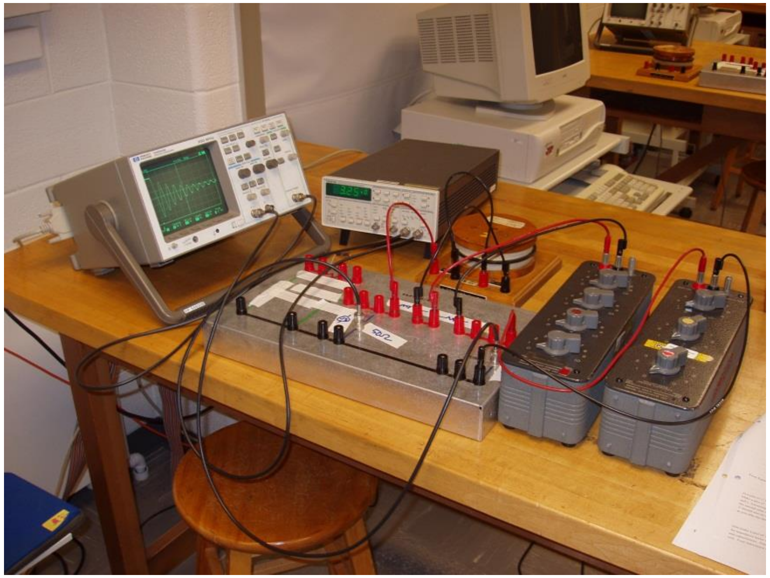
transient (physics) a short-lived oscillation in a system caused by a sudden change of voltage or current or load

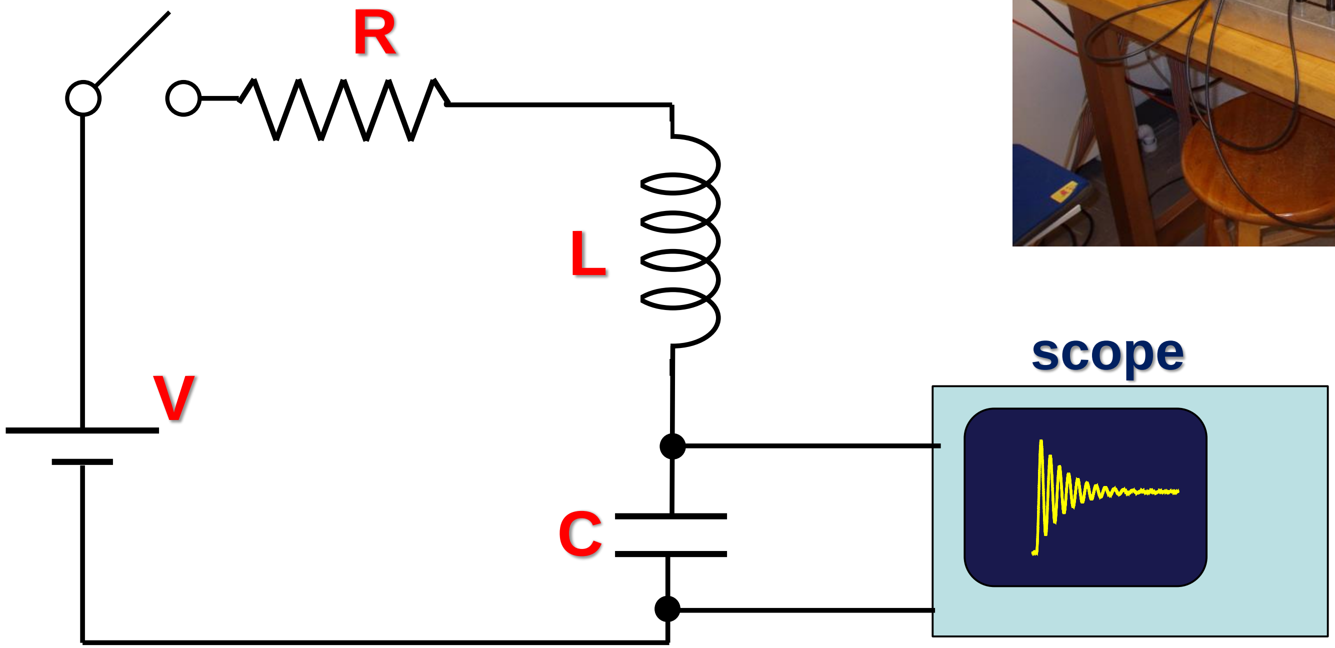
a transient response or natural response is the response of a system to a change from equilibrium.



Transients in RLC circuit.

Resistance R [Ohm]
Capacitance C [μF] (10^{-6}F)
Inductance L [mH] (10^{-3}H)





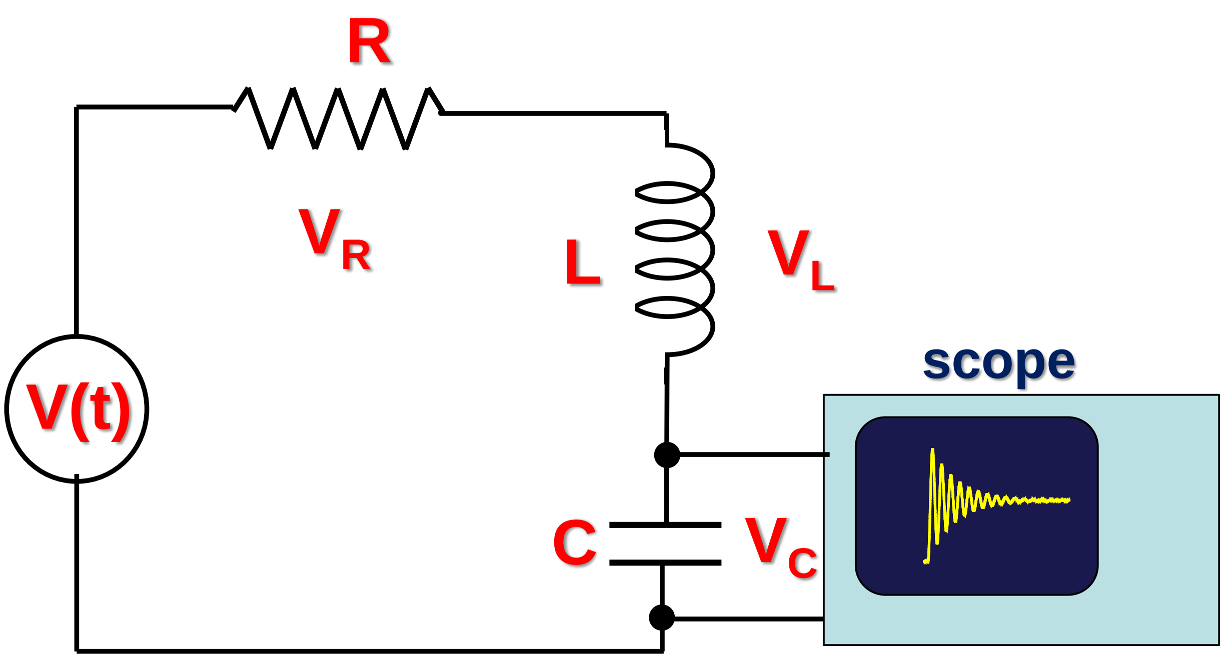
Transients in RLC circuit.

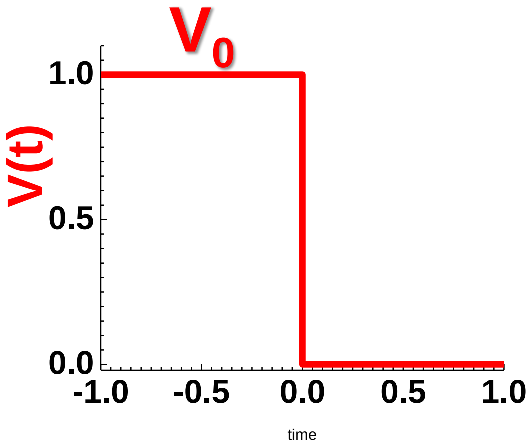
According the Kirchhoff's law $V_R+V_L+V_C=V(t)$

*

$$L\frac{d^2}{dt^2}q(t)+R\frac{d}{dt}q(t)+\frac{q(t)}{C}=0 \qquad (1)$$

*See Lab write-up for details





Transients in RLC circuit. Three solutions.

The solution of this differential equation can be found in the form

$$q(t) = A e^{st}$$

This will convert (1) in quadratic equation

$$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$$

with solutions:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} \equiv -a \pm b$$

$$a = \frac{R}{2L}\ , \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$b^2>0$ over-damped solution
$b^2=0$ critically damped solution
$b^2<0$ under-damped solution

Transients in RLC circuit. Over-damped solution: $b^2>0$

In this case the solution will be aperiodic exponential decay function with no oscillations:

$$\boxed{q(t) = e^{-at}\left(A_1 e^{bt} + B_1 e^{-bt}\right)}$$

$$\boxed{i(t) = \frac{dq}{dt} = -ae^{-at}(A_1 e^{bt} + B_1 e^{-bt}) + be^{-at}(A_1 e^{bt} - B_1 e^{-bt})}$$

$$b^2 > 0 \rightarrow R^2 > \frac{4L}{C}$$

Transients in RLC circuit. Over-damped solution: $b^2>0$

Taken in account the initial conditions: $q(0)=q_0$ and $i(0)=0$

$$q(t) = q_0 e^{-at}\left(\cosh bt + \frac{a}{b}\sinh bt\right)$$

$$\xrightarrow[(a-b)t>>1]{} \qquad \frac{q_0}{2}\left(1+\frac{a}{b}\right)e^{-(a-b)t}$$

$$i(t) = -\frac{q_0}{2}\left(\frac{a^2-b^2}{b}\right)e^{-(a-b)t}$$

This is exponential decay function

Transients in RLC circuit. Critically-damped solution: $b^2=0$

For this case the general solution can be found as

$q(t)=(A_2+B_2t)e^{-at.}$ Applying the same initial condition

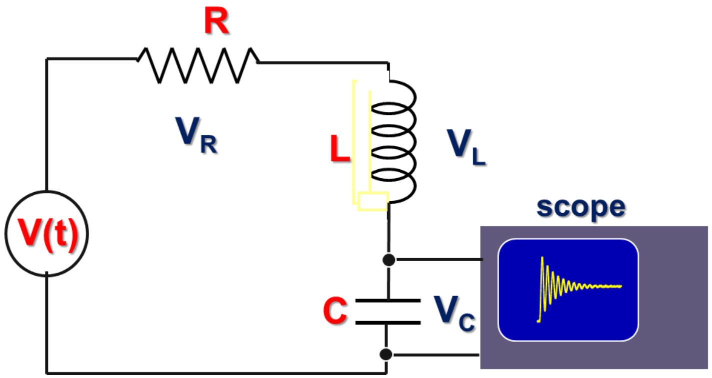
the current can be written as $i=-a^2q_0te^{-at}$

$$b^2 = 0 \rightarrow R^2 = \frac{4L}{C} \quad \text{and} \quad a = \frac{R}{2L}$$

Critically-damped conditions for our network

Critical damped case shows the fastest decay with no oscillations

Transients in RLC circuit. Critically-damped solution: $b^2=0$. Real data analysis.

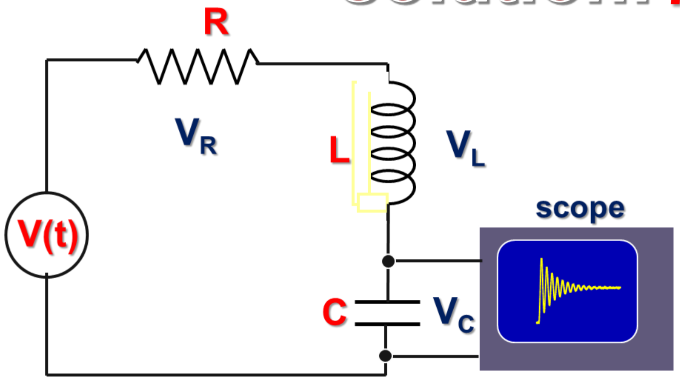


In this experiment R=300 ohms, C=1μF, L=33.43mH.

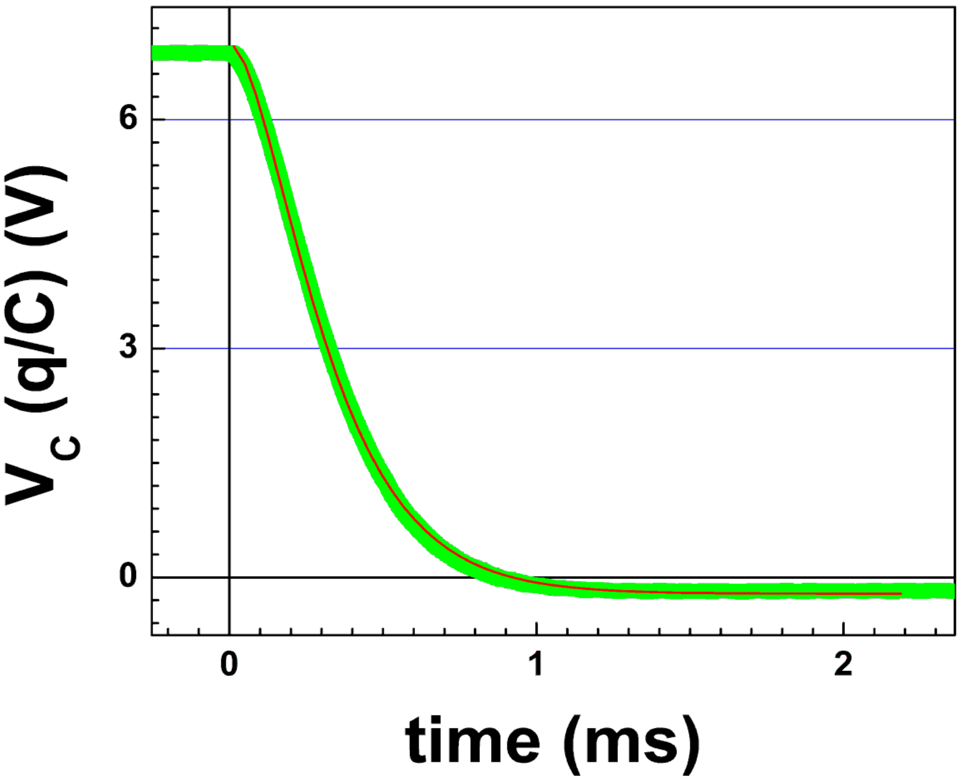
The output resistance of Wavetek is 50 ohms and resistance of coil was measured as 8.7 ohms, so actual resistance of the network is R_a=300+50+8.7=358.7

Decay coefficient $$a = \frac{R}{2L} = \frac{358.7}{2*33.43E-3} \approx 5365$$

Transients in RLC circuit. Critically-damped solution: $b^2=0$. Real data analysis.



Vc ~q, fiiting function: $V_c=V_{co}(1+at)e^{-at}$



Calculated decay coefficient ~5385,

Obtained from fitting - ~5820.
Possible reason – it is still slightly over damped
Calculated b^2 is
$b^2=2.99e7-2.90e7>0$

Transients in RLC circuit. Under-damped solution.

If $b^2<0$ we will have oscillating solution. Omitting the details (see Lab write-up) we have the equations for charge and current as:

$$q(t) = q_0 e^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right) = q_0 e^{-at}\sqrt{1+\frac{a^2}{b^2}}\ \sin(bt+\varphi)$$

$$i(t) = q_0 e^{-at}\left(\frac{a^2+b^2}{b}\right)\sin bt$$

$$a = \frac{R}{2L}\ ,\ b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)};f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$

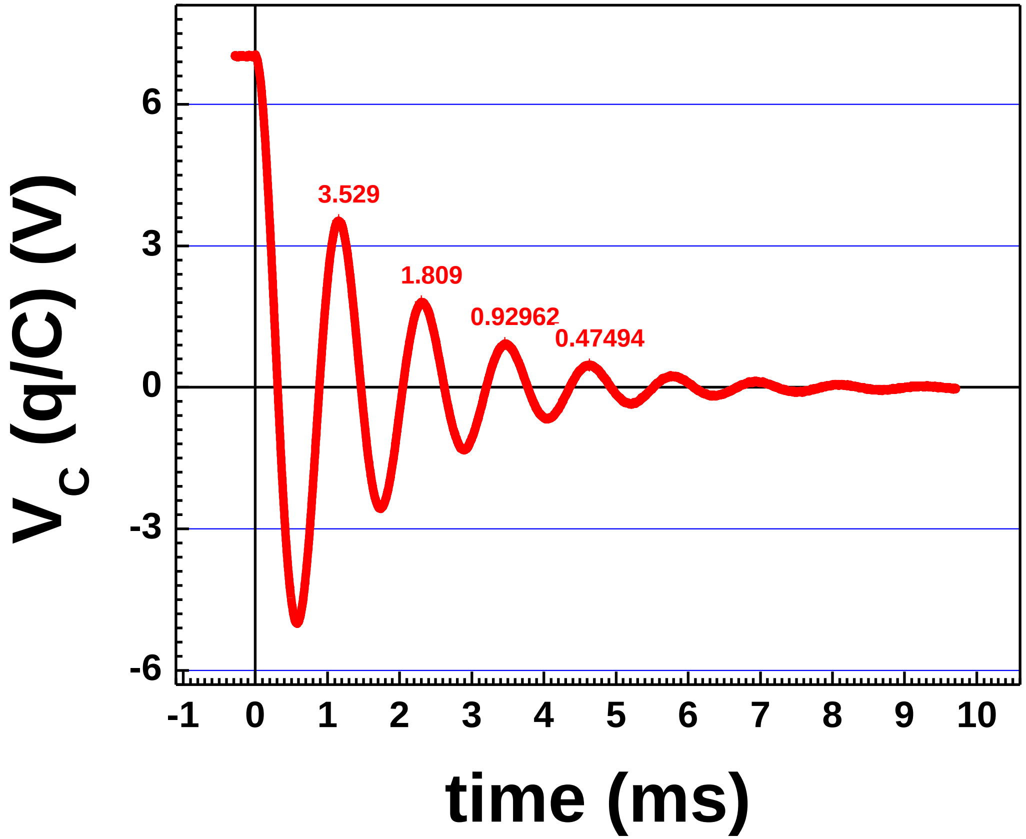
Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

Log decrement can be defined as $\delta = ln\left(\frac{q(t_{max}}{q(t_{max}+T_1)}\right) = ln\left(\frac{e^{-at_{max}}}{e^{-a(t_{max}+T_1)}}\right) = aT_1$, where T_1=1/f1

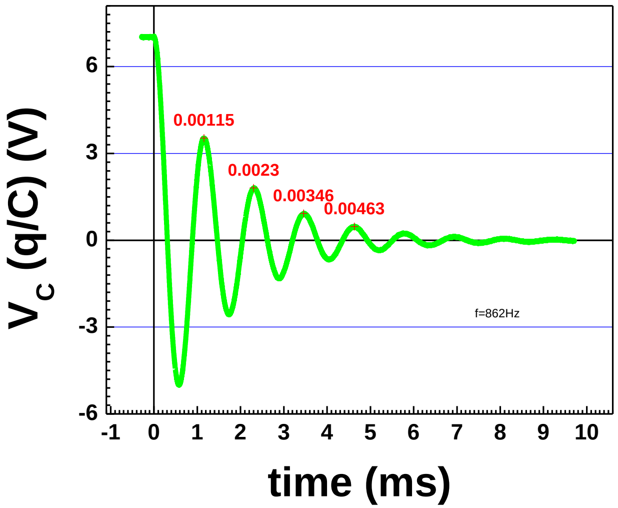
Quality factor can be defined as $Q = 2\pi\frac{E}{\Delta E}$,

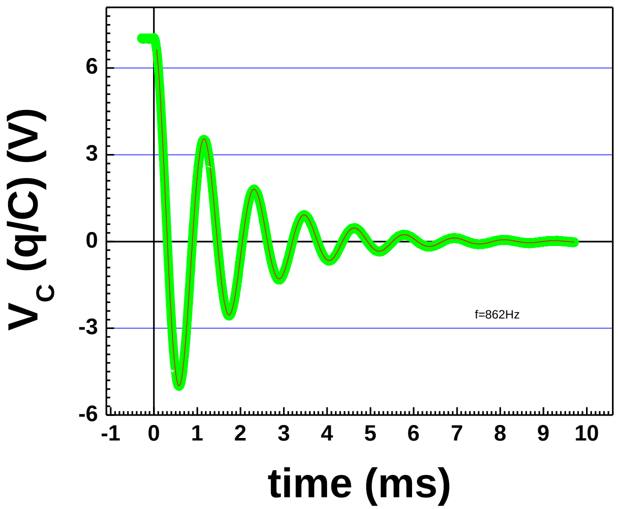
For RLC $Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta}$

From this plot δ≈0.67
Q≈4.7



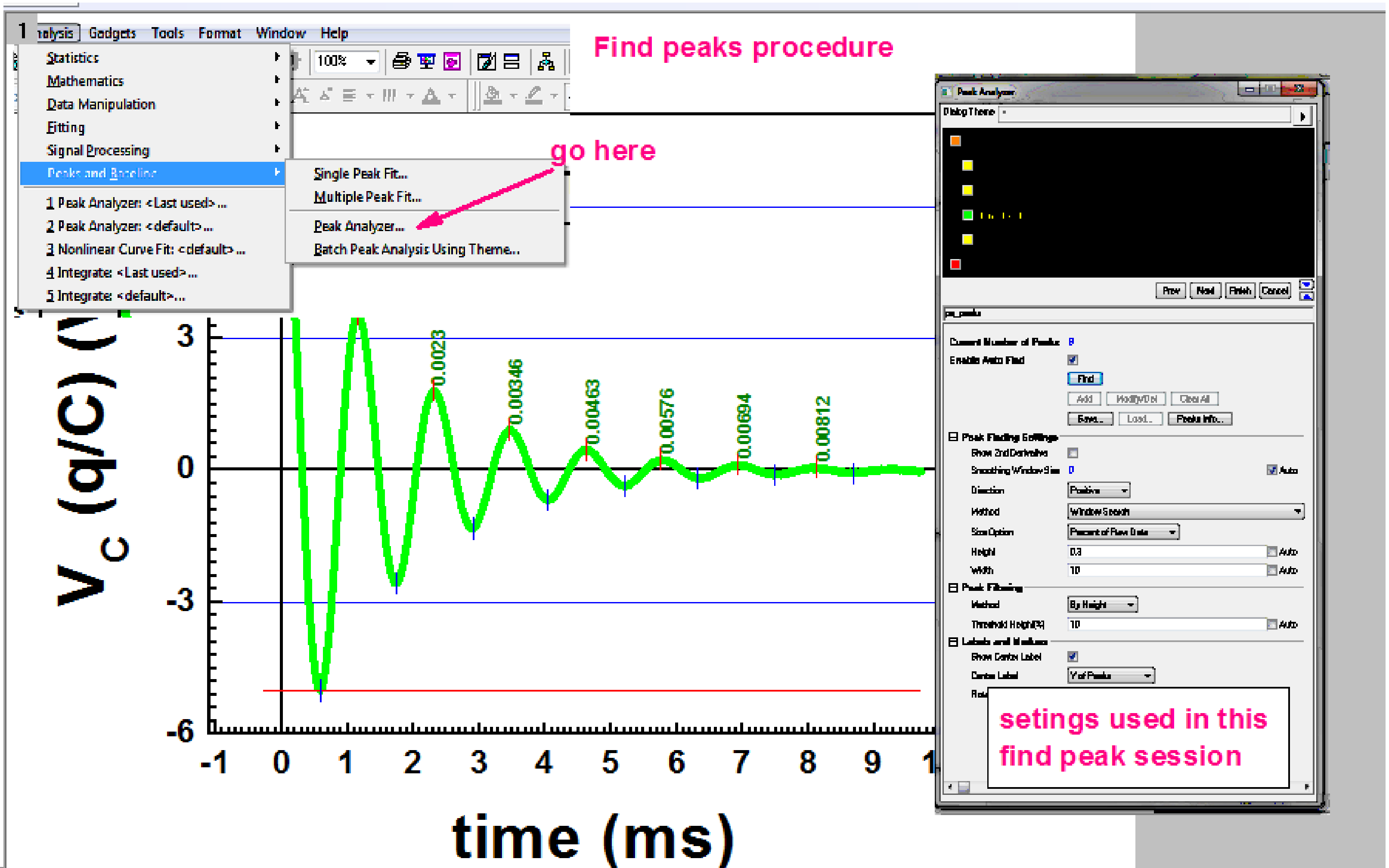
Transients in RLC circuit. Data analysis. Using Origin software.



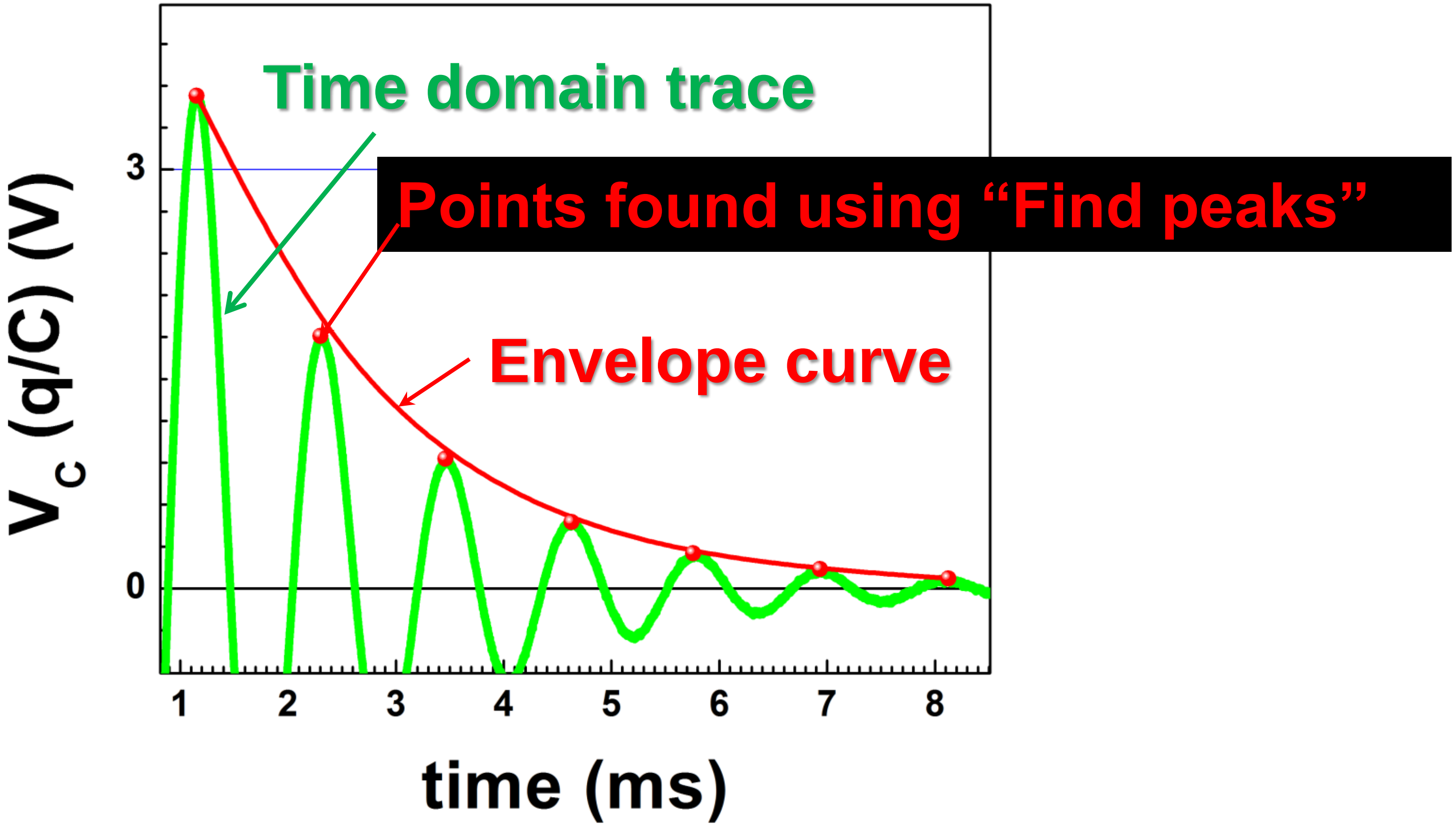


1. **Pick peaks**
2. **Envelope**
3. **Nonlinear fitting**

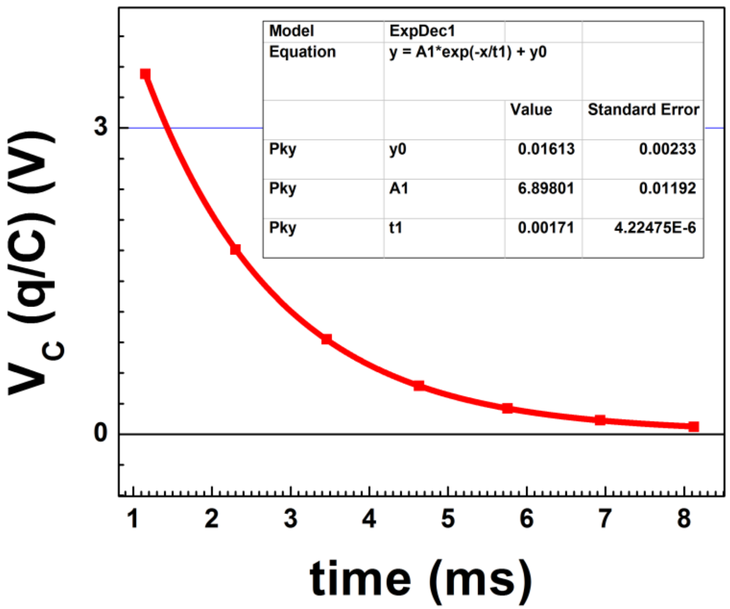
Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

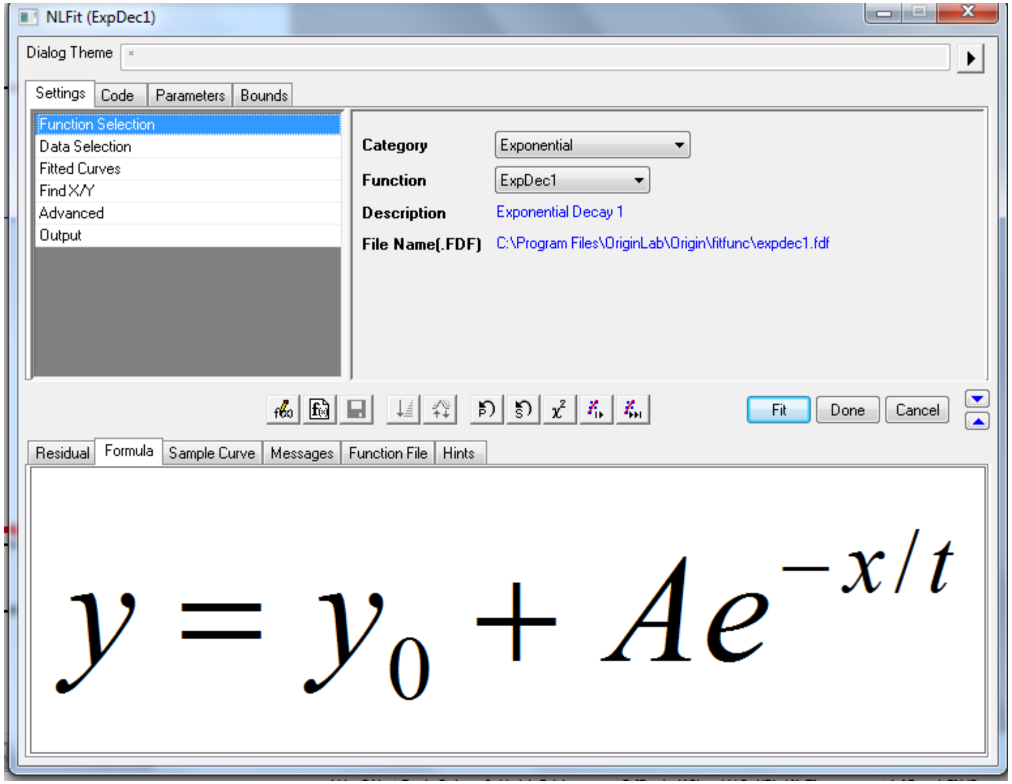


Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.



Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.

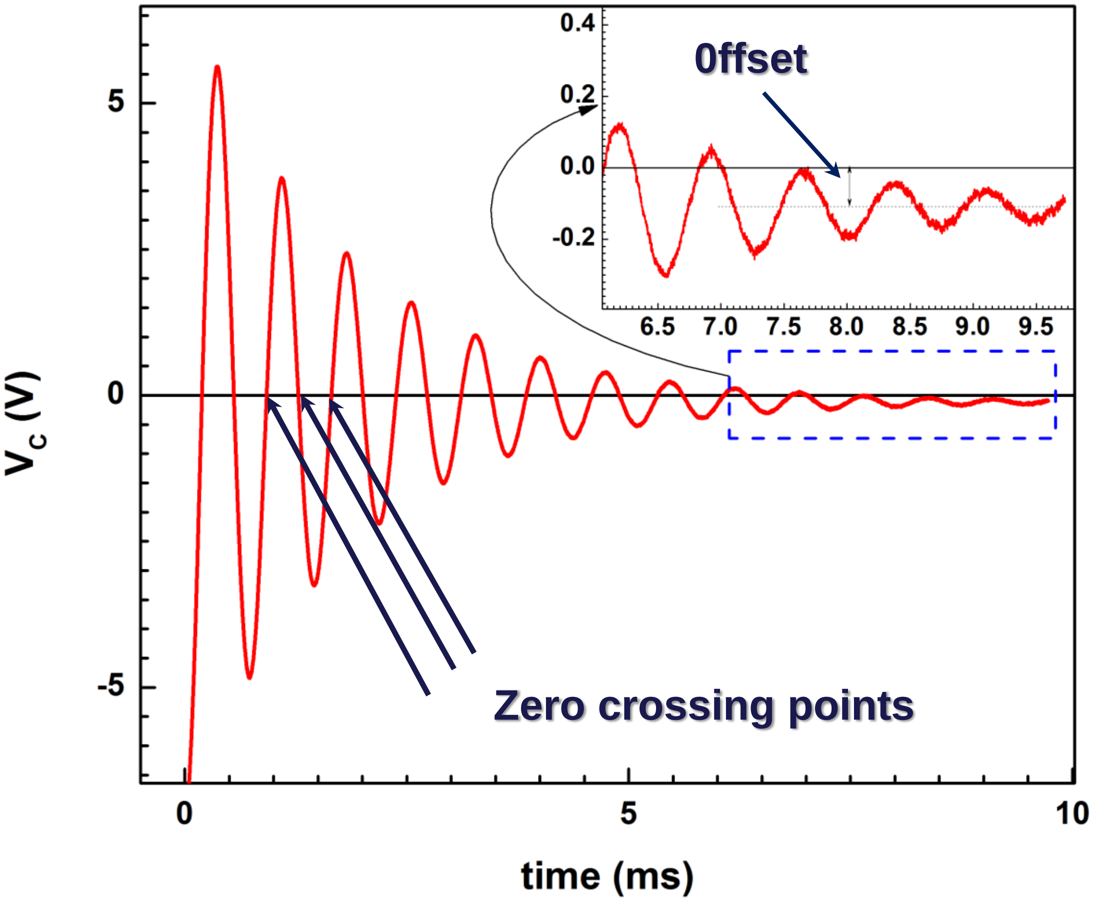




Fitting the "envelope data" to exponential decay function

Transients in RLC circuit. Data analysis. $(1/T)^2$ vs 1/C experiment.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi) + \boxed{offset}$$

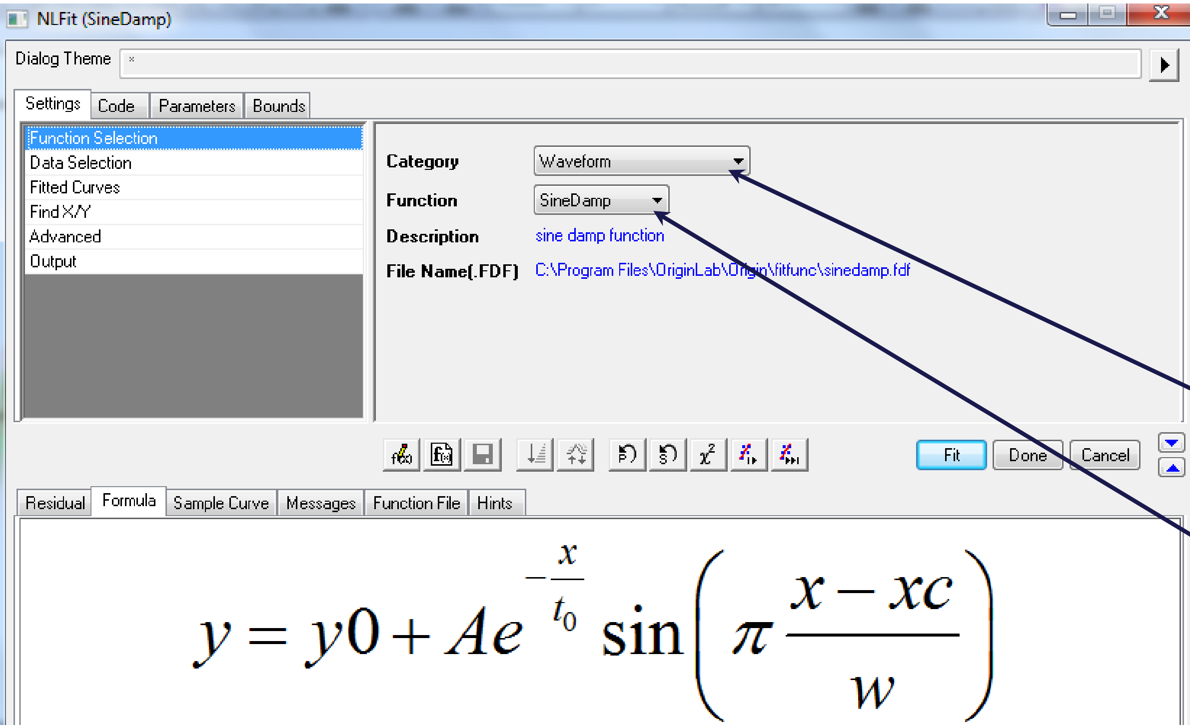


Manual evaluation of the period of the oscillations

Limited accuracy

Results can be effected by DC offset

Transients in RLC circuit. Data analysis. $(1/T)^2$ vs $1/C$ experiment. Using Origin software.



$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

$$U_C = \frac{q(t)}{C}$$

Use Origin standard function

Category: Waveform

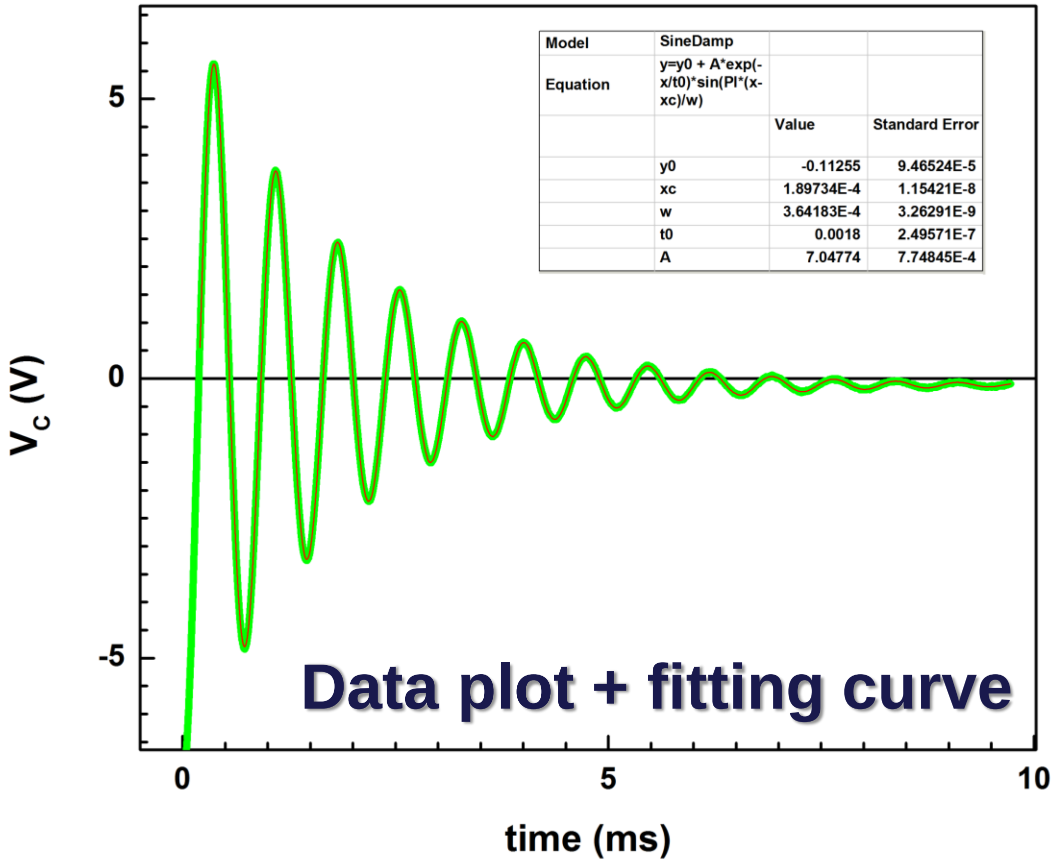
Function: SineDamp

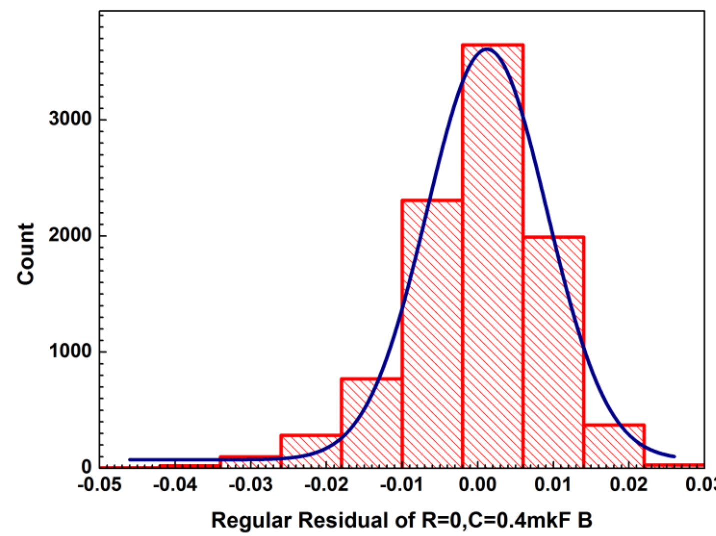
Fitting function ; y0, A, t_0 xc, w – fitting parameters

From fitting you can get: $a = \dfrac{1}{t_0}$ **and** $T = \dfrac{1}{f} = 2w$

Transients in RLC circuit. Data analysis.
$(1/T)^2$ vs 1/C experiment. Using Origin software.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

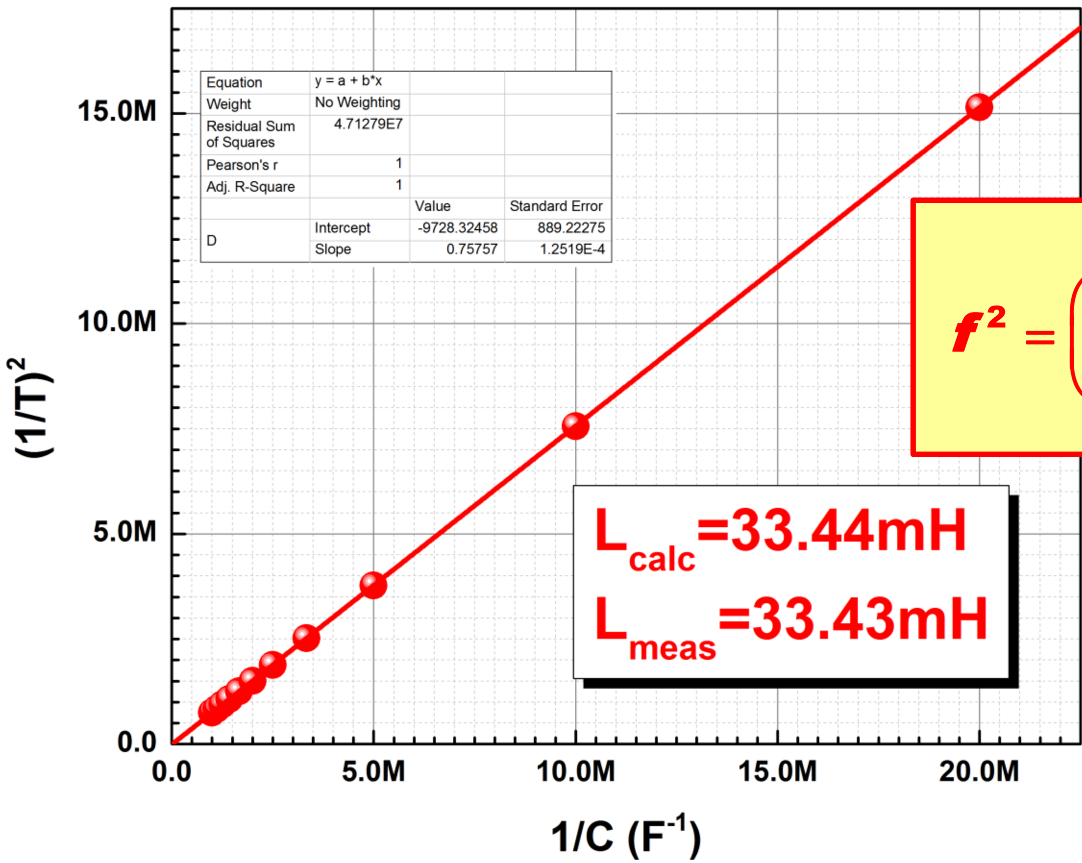




Residuals - criteria of quality of fitting

Transients in RLC circuit. Data analysis. $(1/T)^2$ vs 1/C experiment. Using Origin software.

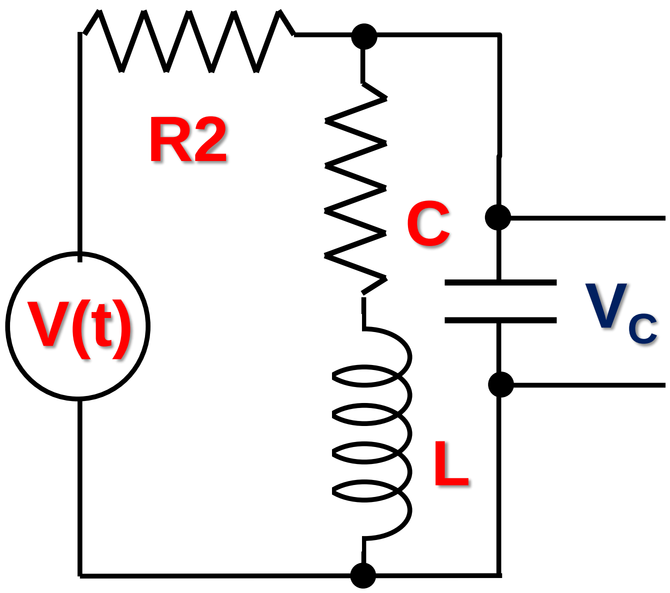
$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

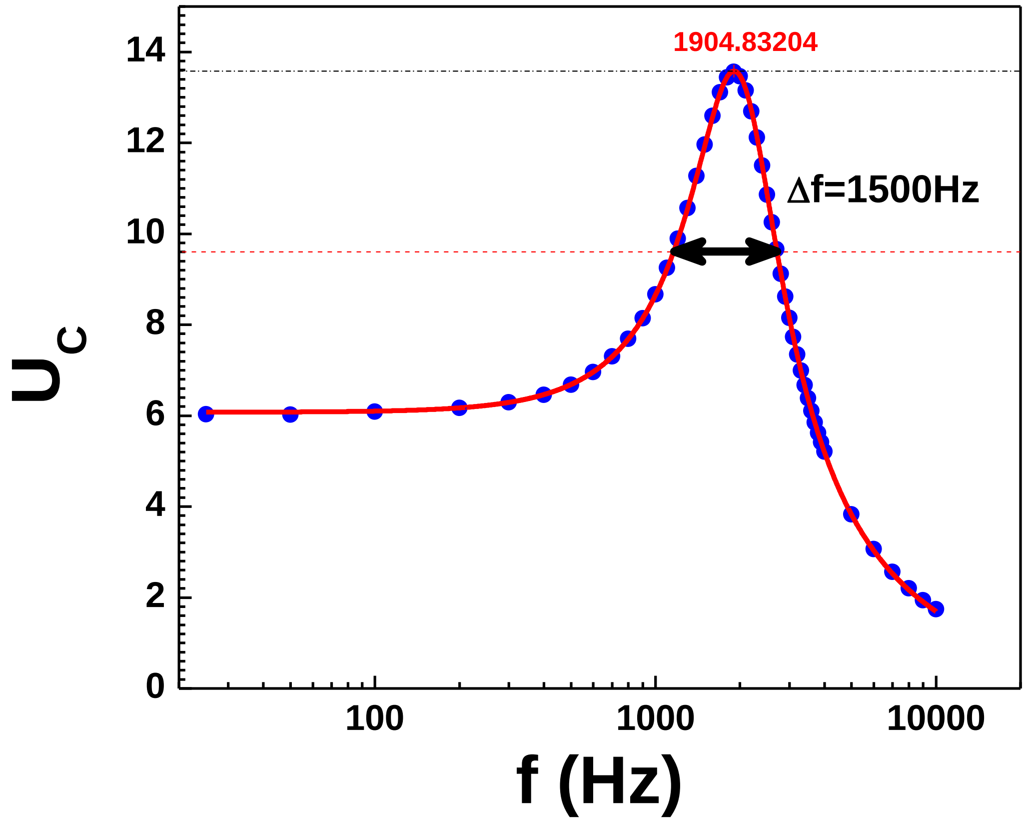


$$f^2 = \left(\frac{1}{T}\right)^2 = \left(\frac{1}{2\pi}\right)^2\left(\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2\right)$$

Final results

Resonance in RLC circuit.





$$Q = \frac{f}{\Delta f} = \frac{1904}{1500} = 1.26$$

Origin templates for this week Lab.

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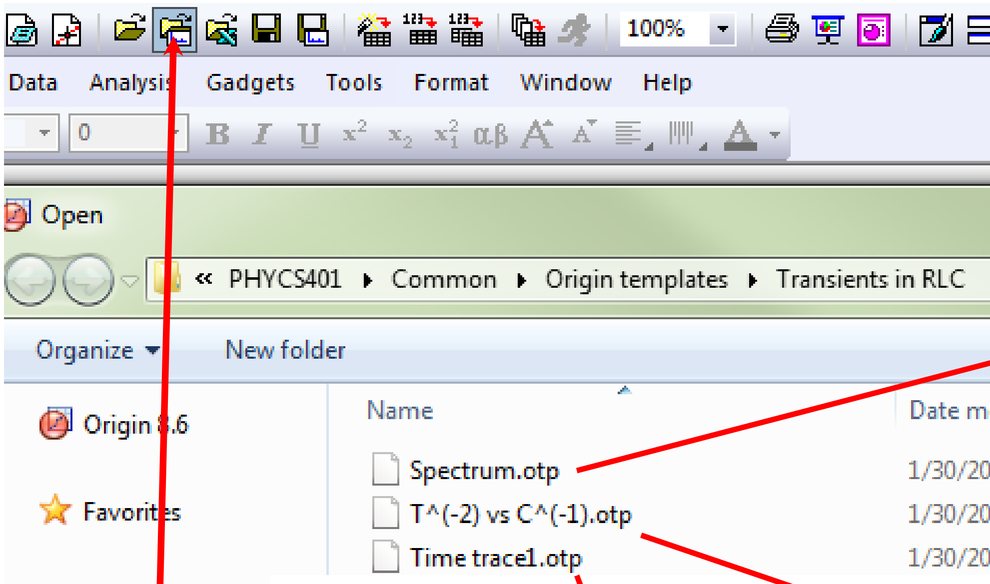
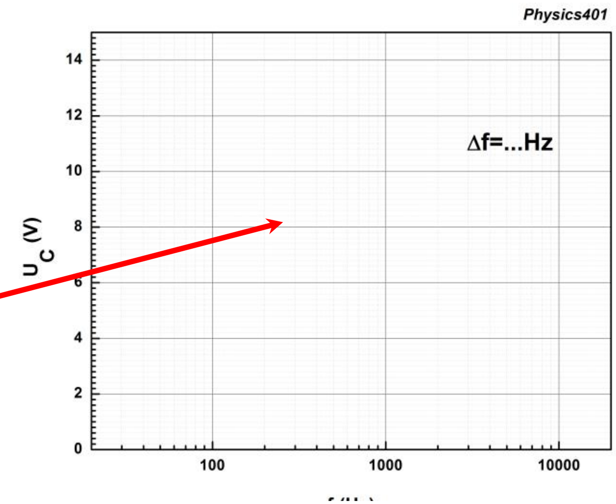
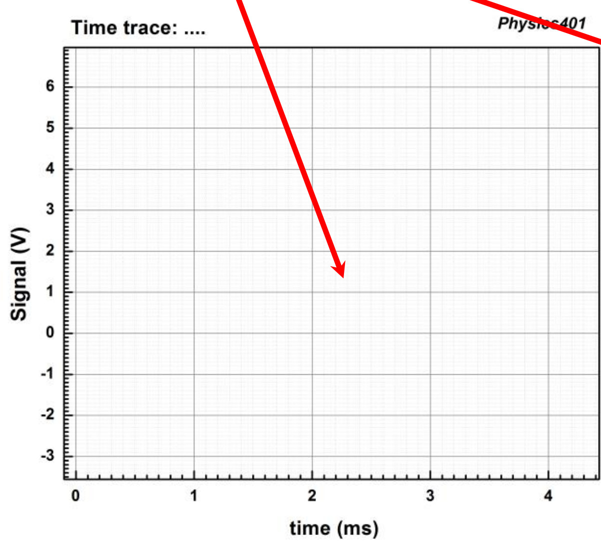
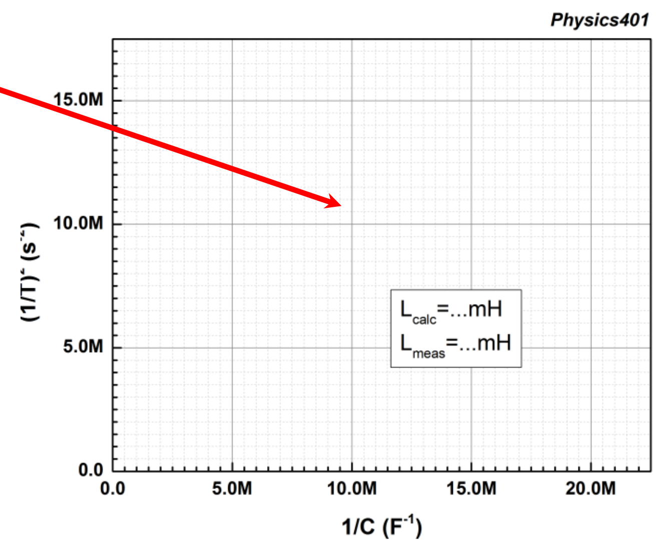




Open template **button**





Origin manuals

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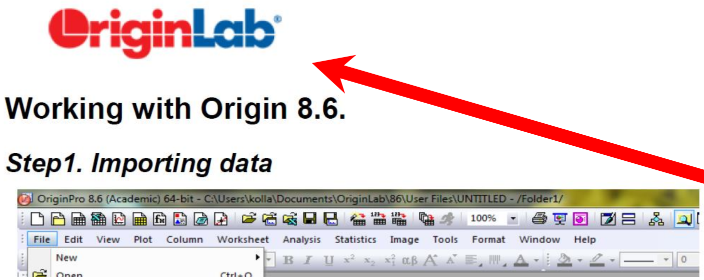
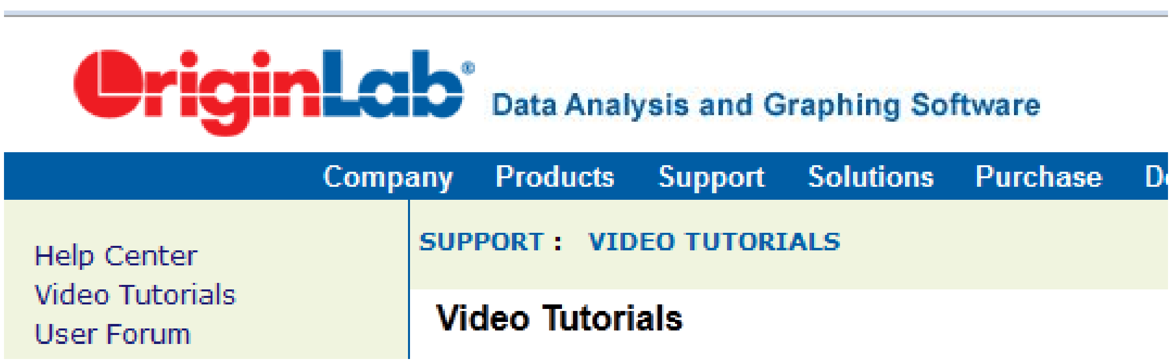


Very short and simple manual which covers only main general operations with Origin. Document located on server and there is a link from P401 WEB page

There are also manuals from OriginLab.

Do not forget about Origin Help

Video Tutorials at the site of the company



http://www.originlab.com/index.aspx?go=SUPPORT/VideoTutorials